E 2447

(Pages : 4)

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2015

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Mathematics for B.C.A.)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define rank of a matrix.
2. Write the system of equations in matrix form $x + z = 1$, $2x - y = 2$, $-2y - z = 0$.
3. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.
4. Find $\lim\limits_{x \to 5} \dfrac{x-5}{x^2-25}$.
5. Does the curve $y = x^3$ ever have a negative slope ? Give reason for your answer.
6. State the mean value theorem.
7. Derive a partial differential equation by eliminating A and B from $z = Ax + By + A^2 + B^2$.
8. Eliminate the arbitrary function f from $z = f\left(x^2 - y^2\right)$.
9. Applying the definition of Laplace transform obtain L $(\sin a\,t)$.
10. Find the inverse Laplace transform of $\dfrac{s^2 - 3s}{s^3}$.

(10 × 1 = 10)

Turn over

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Obtain the normal form of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}.$$

12. Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$.

13. Using matrices test for consistency and then solve the system of equations :

$$2x - y = 5, 3x + 2y = 7.$$

14. Where does the curve $f(x) = \frac{x}{x-1}$ have slope equal to -1 ?

15. Is the function $y = |x|$ differentiable at the origin ? Justify your answer.

16. Find the absolute extreme a of the function $f(x) = x^2$ on $[-2, 1]$.

17. Verify Rolle's theorem for $f(x) = x^2 - 6x + 8$ on $[2, 4]$.

18. Form the partial differential equation by eliminating the arbitrary constants $(x-a)^2 + (y-b)^2 + z^2 = 1$.

19. Eliminate the arbitrary function f from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

20. Find $L(\sin^3 2t)$.

21. Find $L(t^2 e^{-2t})$.

22. Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$.

(8 × 2 = 16)

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the rank of A by reducing it to its canonical form :

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 1 & 2 & -6 & 7 \\ 2 & 4 & 3 & 5 \end{bmatrix}.$$

24. Using matrices find all non-trivial solutions of the system :

$$x_1 - 2x_2 + 3x_3 = 0, 2x_1 + 5x_2 + 6x_3 = 0.$$

25. Find $\lim\limits_{x \to 2} \dfrac{x^3 - 8}{x^4 - 16}$.

26. Does the curve $y = x^3 - 3x - 2$ have any horizontal tangents ? If so where ?

27. Find an equation for the line that is tangent to the curve $y = x^3 - x$ at the point (– 1, 0).

28. Verify the mean value theorem for the function $f(x) = x^2 - 2x + 4$ on [1, 5].

29. Solve the partial differential equation : $(mz - ny)\, P + (nx - lz)\, q = ly - mx$.

30. Find the Laplace transform of $\dfrac{e^{-at} - e^{-bt}}{t}$.

31. Find the inverse Laplace transform of $\dfrac{s}{s^2 + 2s + 5}$.

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Prove that a square matrix A has an inverse if and only if it is non-singular.

(b) Obtain the canonical matrix C row equivalent to the matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 \\ -2 & 1 & 0 & 1 \\ 0 & 1 & 1 & -5 \end{bmatrix}.$$

(c) Use the matrix C in part (b) to check whether the following system of equations is consistent :

$x + z = -1, -2x + y = 1, \; y + z = -5$. If consistent solve the system.

Turn over

33. (a) State Rolle's theorem and interpret it geometrically.

(b) Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution.

(c) Find the velocity and displacement functions of a body falling freely from rest with acceleration 9.8 m/sec^2.

34. (a) Form the partial differential equation by eliminating the arbitrary function f_1 and f_2 from $z = x f_1(x+t) + f_2(x+t)$.

(b) Solve $\frac{\partial^2 z}{\partial y^2} = z$ given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

(c) Solve $(x^2 - y^2 - z^2)p + 2xy\ q = 2xy$.

35. (a) If $L[f(t)] = \bar{f}(s)$, prove that $L\left[\frac{f(t)}{t}\right] = \int_s^\infty \bar{f}(s)\,dS$ provided the integral exists.

(b) Find the Laplace transform of $\frac{\cos 2t - \cos 3t}{t}$.

(c) State convolution theorem and apply it to find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

(2 × 15 = 30)