E 4226 (Pages : 4)

Reg. No..................................

Name..................................

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2016

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Mathematics for B.C.A.)

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define a diagonal matrix.
2. Find all the minors of the matrix $\begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$.
3. Define the normal form of a matrix.
4. Find $\lim_{x \to -2} \sqrt{4x^2 - 3}$.
5. State the sandwich theorem for limit of functions.
6. Derive a partial differential equation by eliminating the constants from the equation :

   $z = ax + by + c$.
7. Define general integral and singular integral of a first order partial differential equation.
8. Find the partial differential equation by eliminating the arbitrary function from $z = f\left(x^2 - y^2\right)$.
9. Using the definition of Laplace transform, find $L(\cosh at)$.
10. State the convolution theorem.

(10 × 1 = 10)

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Using matrices solve the system of equations :

$$2x - 3y - 3 = 0$$
$$4x - y - 11 = 0$$

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12. Find the eigen values and corresponding eigen vectors of the matrix :

$$A = \begin{bmatrix} -2 & -1 \\ 5 & 4 \end{bmatrix}.$$

13. If A and B are any 2 × 2 matrices then show that $(AB)^{-1} = B^{-1}A^{-1}$.

14. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 9$.

15. The curves $y = x^2 + ax + b$ and $y = cx - x^2$ have a common tangent time at the point (1, 0) find a, b and c.

16. Find the absolute maximum and minimum value of the function :

$$f(x) = \frac{-1}{x^2} \text{ where } 0.5 \le x \le 2.$$

17. Form the partial differential equation by eliminating the functions from $z = y f(x) + xy(y)$.

18. Solve $\frac{\partial^2 u}{\partial x \, \partial t} = e^{-t} \cos x$.

19. Find the Laplace transform of $\sin^3 2t$.

20. Find the inverse Laplace transform of $\frac{s^2}{(s-2)^3}$.

21. Apply convolution theorem to evaluate $L^{-1}\left(\frac{s}{s^2 + a^2}\right)$.

22. If $L\{F(t)\} = f(s)$ and $G(t) = \begin{cases} F(t-a) & t > a \\ 0 & t < a \end{cases}$ then prove that $L\{G(t)\} = e^{-as} f(s)$.

(8 × 2 = 16)

Part C (Short Essay Questions)

Answer any **six** *questions.*

Each question carries 4 marks.

23. Reduce the matrix A to its normal form and hence determine its rank where :

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}.$$

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24. Find all the algebraic complements of the matrix :

$$B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -2 \\ 1 & -2 & 2 \end{bmatrix}.$$

25. State and prove Rolle's theorem.

26. An object is dropped from the top of a 100 m high tower. Its height above ground after 1 second is $100 - 4.9\,t^2$. How fast is it falling 2 second after it is dropped.

27. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$ $\frac{\partial z}{\partial x} = a \sin y$ and $\frac{\partial z}{\partial y} = 0$.

28. Solve $(x^2 - yz)\,p + (y^2 - zx)\,q = z^2 - xy$.

29. Find the differential equation of all spheres of fixed radius having their centres lie on the z-axis.

30. Find the Laplace transform of :

$$\frac{\cos at - \cos bt}{t}.$$

31. Find the inverse Laplace transform of :

$$\frac{5s+3}{(s-1)(s^2+2s+5)}.$$

(6 × 4 = 24)

Part D (Long Essay Type Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Consider the system of equations :

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

(a) Check this system for consistency.

(b) Using matrix method solve the above system of equations.

(c) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ then show that : $(A^T)^{-1} = (A^{-1})^T$.

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33. (a) Suppose that f and g are differentiable on [a, b] and that $f(a)=g(a)$ and $f(b)=g(b)$. Show that there is atleast one point between a and b where the tangent to the graphs of f and g are parallel.

(b) Let f be differentiable at every value of x and suppose that $f(1)=1$, that $f^1<0$ on $(-\infty,1)$ and $f^1>0$ on $(1,\infty)$. Show that $f(x)\geq 1$ for all x.

(c) Suppose that f is differentiable on [0, 1] and that its derivative is never zero show that : $f(0)\neq f(1)$.

34. (a) Solve $\left(x^2-y^2-z^2\right)p+2xy\,q=2xz$.

(b) Solve $\frac{\partial^2 z}{\partial x^2}+z=0$ given that when $x=0$, $z=e^y$ and $\frac{\partial z}{\partial x}=1$.

(c) Find the differential equation of all planes which are at a constant distance from the origin.

35. (a) Define Laplace transform of a function. If $L[f(t)]=f(s)$. Prove that $L\left[e^{at}f(t)\right]=f(s-a)$.

(b) Find the inverse Laplace transform of $\frac{s}{s^4+4a^4}$.

(c) State the change of scale properly of the Laplace transform and find $L\left\{\frac{\sin at}{t}\right\}$.

(2 × 15 = 30)

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