E 9892 (Pages : 4) Reg. No..............................

Name..................................

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2014

First Semester

MATRICES, CALCULUS AND LAPLACE TRANSFORMS

(Complementary Mathematics for B.C.A.)

[2013 Admission onwards]

Time : Three Hours

Maximum : 80 Marks

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define non-singular matrix.
2. Define normal form of a matrix.
3. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.
4. If $1 - \frac{x^2}{4} \le u(x) \le 1 + \frac{x^2}{2}$ for all $x \neq 0$, find $\lim_{x \to 0} u(x)$.
5. Find the derivative of $\frac{(x-1)(x^2-2x)}{x^4}$ with respect to x.
6. State Rolle's theorem.
7. Form a partial differential equation by eliminating A and B from $2z = \frac{x^2}{A^2} + \frac{y^2}{B^2}$.
8. Eliminate the arbitrary function f from $z = e^{my} f(x-y)$.
9. By applying the definition of Laplace transform, find $L(\cos at)$.
10. State the convolution theorem.

(10 × 1 = 10)

Turn over

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

12. Using matrices solve the system of equations :

    $2x - 3y = 3, 4x - y = 11.$

13. Find the characteristic equation and the characteristic roots of $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

14. Find the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

15. Find the derivatives of all orders of the function $y = x^4 - 3x^2 + x$.

16. At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$. Find the body's acceleration each time when the velocity is zero.

17. Find the absolute extrema of $f(x) = x^{2/3}$ on $[-2, 3]$.

18. Form the partial differential equation by eliminating the functions f_1 and f_2 from $z = f_1(x) f_2(y)$.

19. Solve $\frac{\partial^2 z}{\partial x^2} = xy$.

20. Find L $(\sin 2t \cos 3t)$.

21. Find $L\left(\cos^3 2t\right)$.

22. Find $L^{-1}\left(\frac{s}{s^2 + 2s + 5}\right)$.

(8 × 2 = 16)

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the rank of the matrix A, by reducing to its normal form :

$$A = \begin{bmatrix} 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \\ 0 & 1 & 2 & -2 \end{bmatrix}.$$

24. Find the inverse of A using only row operations to reduce A to the identity matrix I ;

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

25. Find $\lim\limits_{x \to 1} \dfrac{x-1}{\sqrt{x+3}-2}$.

26. Show that the function $f(x) = |x|$ is differentiable everywhere except at $x = 0$.

27. The curve $y = ax^2 + bx + c$ passes through $(1,2)$ and is tangent to the line $y = x$ at the origin. Find a, b and c.

28. Find the value of c in the mean value theorem for the function $f(x) = \dfrac{x^3}{4} + 1$ on $[0, 2]$.

29. Solve the partial differential equation $\dfrac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0$, $z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$.

30. Find $L^{-1}\left(\dfrac{1-e^t}{t}\right)$.

31. Find the inverse Laplace transform of $\dfrac{s+5}{s^2 - 2s - 3}$.

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions. Each question carries* 15 *marks.*

32. (a) If A and B are any two non-singular matrices of the same order, then prove that AB is non-singular and $(AB)^{-1} = B^{-1} A^{-1}$.

(b) Find the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}.$$

(c) Use A^{-1} in part (b) to solve the system of equations

$$x + 2y + 3z = 3,\ \ 2x + 3y + 2z = 0,\ \ 3x + 3y + 4z = 5.$$

33. (a) State the mean value theorem and interpret it geometrically.

(b) Deduce that if $f'(x) = 0$ for all x in an open interval (a, b), then $f(x) = c$ for all x in (a, b) where c is a constant.

(c) For what values of a, m and b does the function

$$f(x) = \begin{cases} 3 & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b & 1 \le x \le 2 \end{cases}$$

satisfy the hypotheses of the mean value theorem on [0, 2].

34. (a) Find the partial differential equation of all spheres whose centre lie on the z-axis.

(b) Explain the terms complete integral, particular integral, general integral and singular integral of a first order partial differential equation.

(c) Solve $x^2 (y - z)\, p + y^2 (z - x)\, q = z^2 (x - y)$.

35. (a) Define Laplace transform of a function. If $L[f(t)] = \bar{f}(s)$, prove that $L\left[e^{at} f(t)\right] = \bar{f}(s - a)$.

(b) If $L[f(t)] = \bar{f}(s)$, show that $L = [(\sin h\, at) f(t)] = \frac{1}{2}\left[\bar{f}(s-a) + \bar{f}(s+a)\right]$ and $L[(\cos h\, at) f(t)] = \frac{1}{2}\left[\bar{f}(s-a) + \bar{f}(s+a)\right]$. Hence evaluate $L(\sin h\, 2t \sin t)$.

(c) Apply convolution theorem to the inverse Laplace transform of $\frac{1}{\left(s^2 + a^2\right)^2}$.

(2 × 15 = 30)