E 2569 (Pages : 4) Reg. No................................

Name..................................

B.C.A. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2016

Second Semester

DISCRETE MATHEMATICS

(Complementary Course to B.C.A.)

(2013 Admission onwards)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions from this part.*
Each question carries 1 *mark.*

1. Determine the sets A and B, where $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$ and $A \cap B = \{4, 9\}$.
2. Determine whether the relation $\{(x, y)\,(x, y \in Z, y = x^2 + 7)\}$ a function.
3. State the rule of sum.
4. Find the number of words of three distinct letters formed from the letters of the word "JNTU".
5. State the principle of mathematical induction.
6. What is the rule of instantiation ?
7. State rule of chains.
8. What do you mean by the term minimal spanning tree ?
9. Define a planar graph.
10. Define a binary tree.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Using Venn diagram investigate the truth or falsity of the following :—

    For sets $A, B, C \in \mathcal{U}$,

    $A \Delta (B \cap C) = (A \Delta B) \cap (A \Delta C)$.

Turn over

12. Using the laws of set theory, simplify $(A - B) \cup (A \cap B)$.
13. How many arrangements can be made with the letter of the word 'MATHEMATICS'. In how many of them vowels are together.
14. How many numbers between 400 and 1000 can be formed with the digits 0, 2, 3, 4, 5, 6 if no digit is repeated in the same number.
15. Verify the first absorption law by means of a truth table.
16. Use the substitution rules to show that $[p \to (q \vee r)] \Leftrightarrow [(p \wedge \neg q) \to r]$.
17. Determine the truth value of the following implications.
    (a) If $3 + 4 = 12$, then $3 + 2 = 6$.
    (b) If $3 + 3 = 6$, then $3 + 4 = 9$.
18. Let $f = z^+ \to z^+$ defined by $f(x) = x + 1$ and $g : z^+ \to z^+$ by $g(x) = \max\{1, x - 1\}$. Find $g \circ f$.
19. In how many ways can one distribute 10 (identical) white marbles among six distinct containers.
20. Give an example of a connected graph where removing any edge of G results in a disconnected graph.
21. Prove that in any graph G, the number of vertices of odd degrees is even.
22. Show that the Petersen graph P is non-planar.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Two integers are selected at random without replacement from $\{1, 2, ...99, 100\}$. What is the probability that their sum is even.
24. Verify that the relation, two ordered pairs (a, b) and (c, d) are equivalent if $ad = bc$ is an equivalence relation on the set S of all ordered pairs (a, b) of integers with $b \neq 0$.
25. Discuss the method of proof by adopting a premise.
26. What is the minimum distance of a code consisting of the following code words : 001010, 011100, 010111, 011110, 101001.
27. Using the laws of set theory simplify $\bar{A} \cup \bar{B} \cup (A \cap B \cap \bar{C})$.

28. Draw the Hasse diagram for the poset $(P(\mathcal{U}), \subseteq)$, where $\mathcal{U} = \{1, 2, 3, 4\}$.

29. Describe the Konigsberg bridge problem and state its solution using Graph theory.

30. Prove that the number of edges in a tree with n vertices is $n-1$. Also prove that a connected graph with n vertices and $n-1$ edges is a tree.

31. Describe the shortest path problem.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 marks.

32. (a) Prove that ${}^{n}P_r = {}^{n-1}P_{r+r} \cdot {}^{n-1}p_{r-1}$

(b) If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3:5$ find n.

(c) How many different signals can be given using any number of flags from 5 flags of different colours.

33. (i) We truth tables to verify that each of the following is a logical implication.

(a) $[(p \to q) \wedge (q \to r)] \to [p \to r]$.

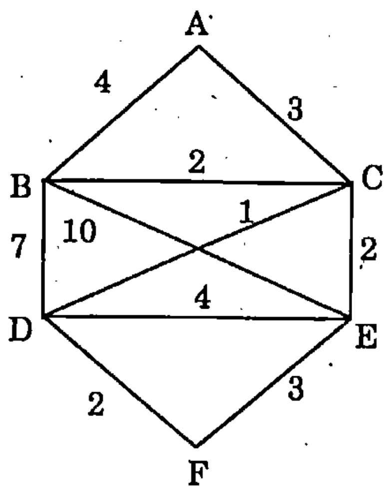
(b) $[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$.

(ii) Let $p(x), q(x)$ denote the following open statements $p(x): x \leq 3$ $q(x): x+1$ is odd. If the universe consists of all integers, what are the truth values of the following statements.

(a) $q(1)$. (b) $\neg p(3)$.

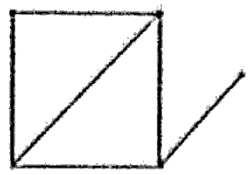
(c) $p(7) \vee q(7)$.

34. We Dijkstra's algorithm to find the shortest path from A to F.



Turn over

35. (a) Determine the number of spanning trees of the following graph :



(b) Given an example of a graph with n vertices and $n-1$ edges that is not a tree.

(c) Describe the spanning tree algorithm.

$(2 \times 15 = 30)$